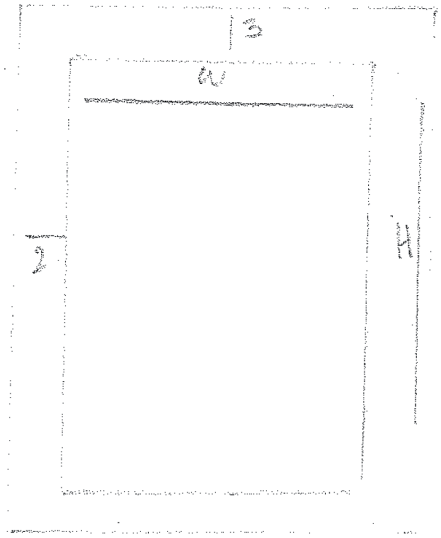
**Title: The Printed Page**




**Introduction**

In my investigation, my aim is to find the dimensions of the printable area in order to make the area of the entire page a minimum. Due to the fact that the printed page must remain with an area of 144cm$^2$, I can only alter the printed page dimensions according to the 144 cm$^2$. In order to do this, first I will need to calculate the area and dimensions of only the printed area, and then I will calculate the dimensions and area of the whole page with my printed area dimensions as a starting point. Due to the fact that the page has margins of 2 cm along each side and 3 cm along the top and bottom, when I want to calculate the height of my entire page, I must add 6cm to the printed area length. When calculating the width of the whole page, I will need to add 4 cm to the printed area width. I will then multiply the total page width with the total page length to find the area of the page, and to find the minimum area of it. The best way to solve this problem is to use the method of trial and error. However, because using this method could take a long time, I will use Microsoft Excel, a programme that will help me calculate the equations I type into it.

A1

Methods:

In order to ensure my answers are correct, I will be using three methods to solve the problem. I will be solving this numerically, algebraically and use calculus.

**Numeric Method:** When solving the equation, the value of the width and length will vary; therefore I will need to follow a method of trial and error. Because the printed page is in the shape of a rectangle, I will therefore need to use a regular formula when calculating the area. The standard equation for finding the area of a rectangle is:

$area = length \times width$

From here I will use trial and error to find my solution. First, I start off with any number; in this case I will be using 9 as my starting number:

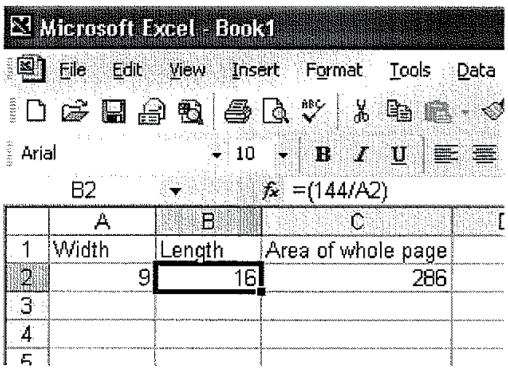
If the width of the printable area (w) is 9, the height of the printable area would be

$\frac{144}{9} = 16$

Now that we have the length and width of the printable area, I am now able to find the area of the whole page

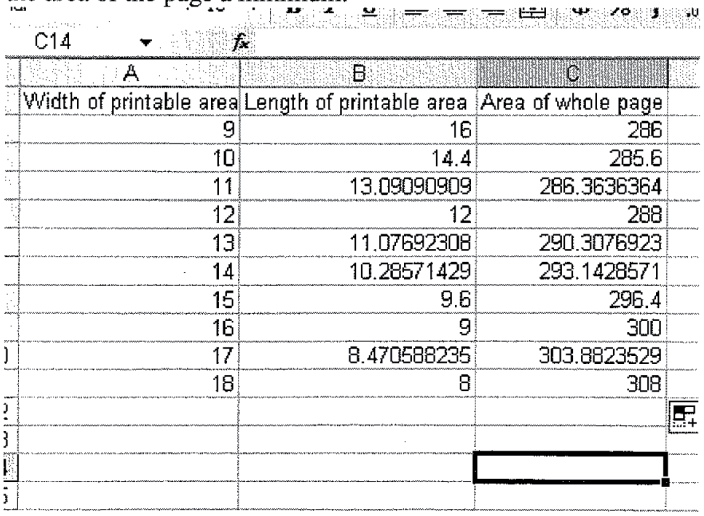
**A= (9+4)(16+6)**
**= 286**

The following table shows how I have solved the problem numerically using Microsoft Excel (with the above example). First of all, I typed in the length of the width and length into the columns, and because I have already figured out my equation for the area of the whole page, I simply had to click on the columns and enter it into the 'fx =' section. For instance, when figuring out what the length of the printable area is, I only needed to insert the digit for the width. Therefore, instead of inserting, $\frac{144}{9} = 16$, I merely had to type in 144 and then click on the column which had consisted of the width.




| | A | B | C |
|---|---|---|---|
| 1 | Width | Length | Area of whole page |
| 2 | 9 | 16 | 286 |
| 3 | | | |
| 4 | | | |

Now I will use Microsoft Excel to find the dimensions of the printable area that make the area of the page a minimum.




| A | B | C |
|---|---|---|
| Width of printable area | Length of printable area | Area of whole page |
| 9 | 16 | 286 |
| 10 | 14.4 | 285.6 |
| 11 | 13.09090909 | 286.3636364 |
| 12 | 12 | 288 |
| 13 | 11.07692308 | 290.3076923 |
| 14 | 10.28571429 | 293.1428571 |
| 15 | 9.6 | 296.4 |
| 16 | 9 | 300 |
| 17 | 8.470588235 | 303.8823529 |
| 18 | 8 | 308 |

2

B2

B2

From this selection of numbers, we can see that the area of the whole page is increasing. However, I am trying to find the minimum area of the page, therefore I will try and decrease the width to find the page area minimum.

| Width of printable area | Length of printable area | Area of whole page |
|---|---|---|
| 1 | 144 | 750 |
| 2 | 72 | 468 |
| 3 | 48 | 378 |
| 4 | 36 | 336 |
| 5 | 28.8 | 313.2 |
| 6 | 24 | 300 |
| 7 | 20.57142857 | 292.2857143 |
| 8 | 18 | 288 |
| 9 | 16 | 286 |
| 10 | 14.4 | 285.6 |
| 11 | 13.09090909 | 286.3636364 |

Here we see that even as I start from the lowest number, 1, the area has actually increased from when I substituted it with 18. However, we start to see a decrease a numbers until I get to 11, where it beings to increase

Now viewing the Microsoft Excel table as a whole:

| A | B | C |
|---|---|---|
| Width of printable area | Length of printable area | Area of whole page |
| 1 | 144 | 750 |
| 2 | 72 | 468 |
| 3 | 48 | 378 |
| 4 | 36 | 336 |
| 5 | 28.8 | 313.2 |
| 6 | 24 | 300 |
| 7 | 20.57142857 | 292.2857143 |
| 8 | 18 | 288 |
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| 12 | 12 | 288 |
| 13 | 11.07692308 | 290.3076923 |
| 14 | 10.28571429 | 293.1428571 |
| 15 | 9.6 | 296.4 |
| 16 | 9 | 300 |
| 17 | 8.470588235 | 303.8823529 |
| 18 | 8 | 308 |
| 20 | 7.2 | 316.8 |
| 26 | 5.538461538 | 346.1538462 |
| 30 | 4.8 | 367.2 |
| 40 | 3.6 | 422.4 |
| 59 | 2.440677966 | 531.7627119 |
| 70 | 2.057142857 | 596.2285714 |
| 80 | 1.8 | 655.2 |
| 90 | 1.6 | 714.4 |
| 100 | 1.44 | 773.76 |
| 112 | 1.285714286 | 845.1428571 |

3

We can see that when the width of the printable area reaches 6, the total area of the page starts to decrease, until it reaches the minimum point of 285.6, where it gradually begins to increase again.

**Algebraic method:** Having solved the problem numerically, I will now try solving it algebraically. By doing this, I can also determine whether my previous solution is correct. In order to come up with an algebraic equation I am going to find individual formulas for each of the variables. For example:

C3

w = width of only the printable area  W = width of entire page
h = height of only the printable area  H = height of entire page

If $w=9$ ; $h = \frac{144}{9}$

$h = 16$

$W = 9+4$

$= 13$

$H = 16+6$

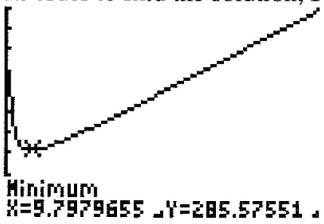
$= 22$

Now that I have used letters to represent the height and width of all the dimensions, I can simplify 'W' and 'H' as:
$W = w+4$; $H = h+4$
As we can see, I still apply the same rules to my algebraic formula. Due to the fact that I must find the area, I can use the letters I have chosen to create a formula for the area:

Area= HW
However, in order to make solving the equation easier, I will only use one letter to express the area. Therefore, the formula becomes:

$A = (w+4)\left(\frac{144}{4}\right)+6)$

In order to find the solution, I must make a graph on my calculator.




As we can see from the above screen capture, I made a graph on my calculator using my algebraic equation. I then used the calculator to find the minimum point on the graph. Studying the screen capture above, it says that the x minimum is 9.7, and if I look back at my numeric tables from Excel, the point at which the width(x) is 9 and 10, is where the minimum area of the entire page is shown to be 285.6. The lowest point on the graph would also be the minimum area to the entire page. Studying the screen capture above, we can see the minimum point on the graph is 285.6 which is the same answer I gathered when I attempted solving it numerically.

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C4

**Calculus**: As I have tried solving the problem both numerically and algebraically, I have drawn to the same conclusion in both methods. Therefore, the dimensions of the printable area which make the area of the entire page a minimum is when the width of the page is 10, and the height of the page is 14.4, thereby making the minimum area of the page 285.6 cm².
After using two separate methods to solve my problem, I will attempt to use calculus to check the accuracy of my previous two answers.

As I have used the equation A= (h+6) (w+4) in my previous solution, I will now expand this equation in order to help me solve the problem using calculus.

$A= (h+6)\ (w+4)$
$= hw+6w+4h+24$
As I am using calculus, I also need to find the constraint in my solution. For instance, because my problem involves the printed matter to remain at 144cm², this would be my constraint. Therefore, the formula for my constraint is:

$hw = 144$

Now after finding my constraint, I am going to rearrange the formula, in order to make $h$ the subject, so my constraint turns into:

$$h=\frac{144}{w}$$

Now that I have found an alternative way to express h, I will substitute this into the above equation (hw+6w+4h+24), so that the equation becomes:

$$A=\left(w\times\frac{144}{w}\right)+6w+\left(4\times\frac{144}{w}\right)+24$$
$$A=144+6w+\frac{576}{w}+24$$
$$A=6w+\frac{576}{w}+168$$

Now that I have found the formula for my area, because I also intend on finding the minimum area of the page, I must find the gradient function, therefore I must make my equation equal to 0.

G1

$$\frac{dA}{dW}=6-\left(\frac{576}{w}\text{ ^ }2\right)$$

minimum dA/dW=0
6-(576÷w²)=0
6=576÷w²=0
Now I must find w, and in order to do this, I must rearrange my equation so that w will be the subject of the equation:
w²=576÷6
=96

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$w=\sqrt{96}$
$w=\pm\sqrt{96}$
$= -9.8 \text{ or } 9.8$

**Conclusion**: Judging from my answer the minimum is 9.8, which can be rounded up to 10. Now if I take a look at my previous solutions to the problem, we can see that when the minimum area is 285.6 the width of my printable area is 10, which is the answer I gathered from my gradient function equation, therefore I am certain that my equation is correct, as I have reached the same conclusion when solving my problem numerically, algebraically and using calculus.

G1

D2

Now that I have solved the problem, I can compare my problem to real life situations. Because my problem involves printed page, I can easily use regular paper to test and confirm my results. I have taken two separate pieces of paper, one placed behind the other to represent the entire page, and a smaller sheet of paper in the middle to represent only the printable area, and the measurements all resulted to the same conclusions as my previous three methods, therefore I can infer that the solution to my problem is correct.

E0

6